Journal of Sound and Vibration (1997) **206**(1), 103–108

LETTERS TO THE EDITOR

IDENTIFICATION OF RESTORING FORCES IN NON-LINEAR VIBRATION SYSTEMS BASED ON NEURAL NETWORKS

Y.-C. Liang

Department of Mathematics, Jilin University, Changchun, 130023, *People's Republic of China*

C.-G. Zhou

Department of Computer Science, Jilin University, Changchun, 130023, *People's Republic of China*

and

Z.-S. Wang

Open Laboratory of CAD/CAM Technology for Advanced Manufacturing, Academia Sinica, Shenyang, 110003, *People's Republic of China*

(*Received* 20 *June* 1995, *and in final form* 2 *April* 1997)

1. introduction

It is very important to identify the model of a dynamic system in the applied mechanics field. With the development of computer technology and the enhancement of experimental means, the identification of the dynamic model has developed rapidly. A vast literature now exists on this topic and a wide variety of techniques is now available, such as parametric or non-parametric identification methods, time domain or frequency domain estimation approaches, etc. But there exist some unavoidable limitations in most of the methods, including that *a priori* information about the system under investigation is required, that the properties of the identified system is constrained and that the nature of the excitation source to be used is restricted, and so forth.

Artificial neural network models have several inherent properties which distinguish them from traditional computational models, such as parallel architectures and computations, higher degree of robustness or fault tolerance and property of adaptation or learning etc. These properties make neural networks the ideal choice in cases in which real-time adaptation and fast processing of large amounts of data are required. For this reason a lot of attention has been paid to neural networks for system identification. But there is a paucity of publications in the open literature that address the use of neural networks for identifying physical systems encountered in the applied mechanics field. Masri *et al*. explored a procedure based on neural networks for the identification of non-linear dynamic systems [1]. It was a successful attempt to employ neural network techniques to physical systems in the applied mechanics field. Because of the abilities of learning and generalization of neural networks, no *a priori* information about the system under investigation is required and the nature of the excitation source to be used is not restricted in the procedure. The approach can be used effectively for the identification of the restoring forces of some typical non-linear structural systems, but it only dealt with the identification of single-degree-of-freedom systems in reference [1]. This study extends the procedure to multi-degree-of-freedom non-linear vibration systems and makes the procedure capable of extensive application.

0022–460X/97/360103 + 06 $25.00/0/sv971084

2. IDENTIFIED MODEL

A general mechanical structure can be discretised into a lumped parameter, n-degree-of-freedom system. The equations of motion can be written as

$$\begin{cases} m_1\ddot{y}_1 + g_1(y_1, \dot{y}_1, \ldots, y_n, \dot{y}_n) = F_1(t), \\ m_2\ddot{y}_2 + g_2(y_1, \dot{y}_1, \ldots, y_n, \dot{y}_n) = F_2(t), \\ \qquad\qquad\vdots \\ m_n\ddot{y}_n + g_n(y_1, \dot{y}_1, \ldots, y_n, \dot{y}_n) = F_n(t), \end{cases} \tag{1}$$

where $g_i(y_1, \dot{y}_1, \ldots, y_n, \dot{y}_n)$ and $F_i(t)$ represent the linear and non-linear restoring force and the input excitation acting at the mass m_i, respectively. It is assumed that the excitations $F_i(t)$ and the accelerations $\ddot{y}_i(t)$ $(i = 1, 2, \ldots, n)$ of the system are available from measurements, and that the mass m_i $(i = 1, 2, \ldots, n)$ are known or easily estimated. The non-linear characteristics of the system and the restoring forces $g_i(y_1, \dot{y}_1, \ldots, y_n, \dot{y}_n)$ $(i = 1, 2, \ldots, n)$ acting on the system are not known. The purpose of the paper is to identify the restoring forces, which are the functions of the displacements and the velocities, using measurements and neural networks.

From equation (1) the restoring forces can be written as

$$g_i(y_1, \dot{y}_1, \ldots, y_n, \dot{y}_n) = F_i(t) - m_i\ddot{y}_i(t), \qquad (i = 1, 2, \ldots, n). \tag{2}$$

The estimation procedure described in this paper requires the velocity and displacement responses simultaneously at each response location. The displacements and the velocities of the system can be found by direct measurements or through integration of $\ddot{y}_i(t)$. When employing an integration procedure the reconstruction of the required signals can be accomplished by integrating the measured acceleration response at each location by using the trapezium rule [2, 3]. There exist two problems in using the integration technique. One is that the integration procedure will introduce a constant of integration at each step and the resultant signal will be

$$\dot{y}_i(t) = \int \ddot{y}_i(t)\,\mathrm{d}t + A_i, \qquad y_i(t) = \int \dot{y}_i(t)\,\mathrm{d}t + A_i t + B_i. \tag{3}$$

It is clear that the velocity and displacement data will suffer from the introduction of mean level and linear drift values, respectively. Using these signals directly in the estimation procedure will give poor estimates. These effects can be removed by passing the integrated signals through a high-pass filter [2–4]. It has been demonstrated that the preprocessed signals can be used successfully in the parameter estimation procedure [2]. Another problem is that if the measured acceleration signal is noisy, then the reconstruction of the velocity and displacement signals could be problematic. If the reconstructed data from the noisy measured data are employed, the identified results must be subject to error. The size of the error appears to increase as noise level increases. Nevertheless, the identification studied in this paper is just characteristic but not parametric, therefore, the effect of noise on the identified restoring forces is less than that on the estimated parameters. A reasonably identified restoring force can still be characterized so long as the measured signals are within acceptable experimental error.

If the displacement, velocity, acceleration and the input excitation signals are taken at discrete times t_k,

$$y_{ik} = y_i(t_k), \quad \dot{y}_{ik} = \dot{y}_i(t_k), \quad \ddot{y}_{ik} = \ddot{y}_i(t_k), \quad F_{ik} = F_i(t_k), \qquad (i = 1, 2, \ldots, n), \tag{4}$$

then the values of the restoring forces at times t_k are

$$g_{ik} = F_{ik} - m_i\ddot{y}_{ik}, \qquad (i = 1, 2, \ldots, n). \tag{5}$$

3. IDENTIFICATION PROCEDURE

A three-layer feedforward perception is employed in this paper. The inputs to the net are the measured or calculated displacements and velocities $y_1(t), \dot{y}_1(t), \ldots, y_n(t), \dot{y}_n(t)$. The network outputs $g_i'(y_1, \dot{y}_1, \ldots, y_n, \dot{y}_n)$ $(i = 1, 2, \ldots, n)$ are the unknown restoring forces. The network topology is represented by the weight matrices $[\mathbf{W}^i]$ and the threshold vectors $\{\boldsymbol{\theta}^i\}$ $(i = 1, 2)$.

Let

$$\{\mathbf{y}\} = \{y_1, \dot{y}_1, \ldots, y_n, \dot{y}_n\}^{\mathrm{T}} \tag{6}$$

be the input vector to the net,

$$\{\mathbf{g}'\} = \{g_1', g_2', \ldots, g_n'\}^{\mathrm{T}} \tag{7}$$

be the output vector to the net,

$$f(x) = \tanh(\alpha x) \quad (\alpha > 0) \tag{8}$$

be a non-linear *activation* function.

The outputs of the network are computed according to the following equations:

$$\begin{aligned} \{\bar{\mathbf{v}}\} &= [\mathbf{w}^1]\{\mathbf{y}\} + \{\boldsymbol{\theta}^1\}, \qquad v_j = f(\bar{v}_j) \quad (j = 1, 2, \ldots, l), \\ \{\bar{\mathbf{g}}\} &= [\mathbf{w}^2]\{\mathbf{v}\} + \{\boldsymbol{\theta}^2\}, \qquad g_j' = f(\bar{g}_j) \quad (j = 1, 2, \ldots, n), \end{aligned} \tag{9}$$

where l is the number of the neurons in the hidden layer.

The identification approach consists of two phases: the network training (or learning) phase and the validation phase. During the training phase, the network is presented with the sequence of input vectors $\{y_{1k}, \dot{y}_{1k}, \ldots, y_{nk}, \dot{y}_{nk}\}^{\mathrm{T}}$ and the sequence of desired output vectors $\{g_{1k}, g_{2k}, \ldots, g_{nk}\}^{\mathrm{T}}$. Given a set of weights and thresholds (which initially is chosen randomly), the input vector is propagated forward through the net and the network output $\{g_{1k}', g_{2k}', \ldots, g_{nk}'\}^{\mathrm{T}}$ is calculated according to equations (9).

The error between the actual system output and the desired output is defined as

$$E = \frac{1}{2}\sum_{k=1}^{p}\sum_{i=1}^{n}(g_{ik} - g_{ik}')^2, \tag{10}$$

where p is the number of patterns in the training set.

The purpose of the training phase is to adjust the weights and thresholds w_{ij}^1, w_{jk}^2, θ_j^1, $\theta_k^2 (i = 1, 2, \ldots, 2n; j = 1, 2, \ldots, l; k = 1, 2, \ldots, n)$ which are the elements of $[\mathbf{w}^1]$, $[\mathbf{w}^2]$, $\{\boldsymbol{\theta}^1\}$ and $\{\boldsymbol{\theta}^2\}$, respectively, in the direction that will reduce the error. The training is performed by the back-propagation algorithm [5].

During the validation phase, the network is given other input vector sequences $\{y_{1\alpha}, \dot{y}_{1\alpha}, \ldots, y_{n\alpha}, \dot{y}_{n\alpha}\}^{\mathrm{T}}$ not among those used for training. If the training was successful and the network is a good identifier, it should produce an output sequence $\{g_{1\alpha}', \ldots, g_{n\alpha}'\}^{\mathrm{T}}$ very close to the actual system output

$$\{g_{1\alpha}, \ldots, g_{n\alpha}\}^{\mathrm{T}} = \{g_1(y_{1\alpha}, \ldots, \dot{y}_{n\alpha}), \ldots, g_n(y_{1\alpha}, \ldots, \dot{y}_{n\alpha})\}^{\mathrm{T}} \tag{11}$$

4. SIMULATED EXAMPLE

In order to verify the neural network procedure extended to multi-degree-of-freedom nonlinear systems, the restoring force identification of a three-degree-of-freedom vibration system with hardening springs is examined. The restoring forces in equations (1) are

$$\begin{cases} g_1(y_1, \dot{y}_1, y_2, \dot{y}_2, y_3, \dot{y}_3) = k_{11}y_1 + k_{12}(y_1 - y_2) + k_{13}(y_1 - y_3) + k_{11}^{(3)}y_1^3 \\ \qquad + c_{11}\dot{y}_1 + c_{12}(\dot{y}_1 - \dot{y}_2) + c_{13}(\dot{y}_1 - \dot{y}_3), \\ g_2(y_1, \dot{y}_1, y_2, \dot{y}_2, y_3, \dot{y}_3) = -k_{12}(y_1 - y_2) + k_{23}(y_2 - y_3) - c_{12}(\dot{y}_1 - \dot{y}_2) + c_{23}(\dot{y}_2 - \dot{y}_3), \\ g_3(y_1, \dot{y}_1, y_2, \dot{y}_2, y_3, \dot{y}_3) = -k_{13}(y_1 - y_3) - k_{23}(y_2 - y_3) + k_{33}y_3 - c_{13}(\dot{y}_1 - \dot{y}_3) \\ \qquad - c_{23}(\dot{y}_2 - \dot{y}_3) + c_{33}\dot{y}_3, \end{cases} \tag{12}$$

where the values of the physical parameters are taken as: $m_1 = 1$ kg, $m_2 = 1{\cdot}3$ kg, $m_3 = 2$ kg, $k_{11} = 1000$ N/m, $k_{12} = 2000$ N/m, $k_{13} = 800$ N/m, $k_{22} = 1200$ N/m, $k_{23} = 1500$ N/m, $k_{33} = 3000$ N/m, $c_{11} = 20$ Ns/m, $c_{12} = 15$ Ns/m, $c_{13} = 10$ Ns/m, $c_{22} = 15$ Ns/m, $c_{23} = 30$ Ns/m, $c_{33} = 25$ Ns/m, $k_{11}^{(3)} = 1\,000\,000$ N/m^3.

The method under consideration imposes no restrictions on the nature of the excitation source to be used as a probing signal. In the present example, the excitation used for training the neural network is a swept sine signal with amplitude 40 N and excitation frequency 2π. The excitation only exists at m_1. The net inputs $\{y_{1k}, \dot{y}_{1k}, y_{2k}, \dot{y}_{2k}, y_{3k}, \dot{y}_{3k}\}^T$ and the desired outputs $\{g_{1k}, g_{2k}, g_{3k}\}^T$ are sampled in the time interval [0·2, 20] s at intervals of 0·2 s. The number of patterns in the sample set is $p = 200$. The number of the hidden layer neurons is $l = 13$. Since the network has 6 inputs, 13 nodes in the hidden layer, and 3 outputs, the total number of the network parameters to be adjusted is 133 (117 weights and 16 thresholds terms). The time required in the training phase depends on the desired error E defined in equation (9). In the present example, the initial error is $E = 35{\cdot}2376$, the desired error is taken as $E = 0{\cdot}0004$, and the number of iterations of the present neural network during the training phase is 10 379. The data used in the validation phase are obtained by employing the excitation at m_1 with amplitude 38 N. The reaction speed in the validation phase is very fast. The identification results can be obtained immediately after inputting the validation data into the network. Table 1 gives the minimum and maximum values of measurements and identifications of displacements, velocities and

TABLE 1

Comparisons between measurements and identifications of displacements, velocities and restoring forces

Measurements	Minima	Maxima	Estimates	Minima	Maxima	Errors of minima (%)	Errors of maxima (%)
y_1	−0·01426	0·01418	y_1'	−0·01364	0·01355	4·35	4·44
$\dot{y}_1$	−0·07850	0·10188	$\dot{y}_1'$	−0·06898	0·09680	12·13	4·99
y_2	−0·00755	0·00744	y_2'	−0·00722	0·00711	4·37	4·44
$\dot{y}_2$	−0·04243	0·05394	$\dot{y}_2'$	−0·04026	0·05125	5·11	4·99
y_3	−0·00433	0·00434	y_3'	−0·00414	0·00415	4·39	4·38
$\dot{y}_3$	−0·02388	0·03152	$\dot{y}_3'$	−0·02282	0·02993	4·44	5·04
g_1	−37·68837	39·23965	g_1'	−35·84920	37·28506	4·88	4·98
g_2	−0·39631	0·32541	g_2'	−0·33963	0·30124	14·30	7·43
g_3	−0·38574	0·27437	g_3'	−0·35752	0·25204	7·32	8·14

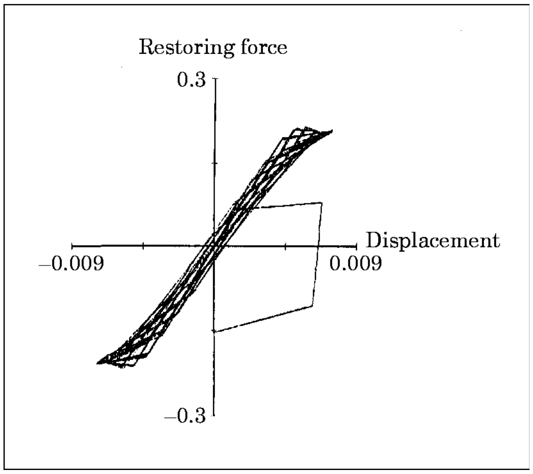

Figure 1. Measurements $g_2(y_2)$.

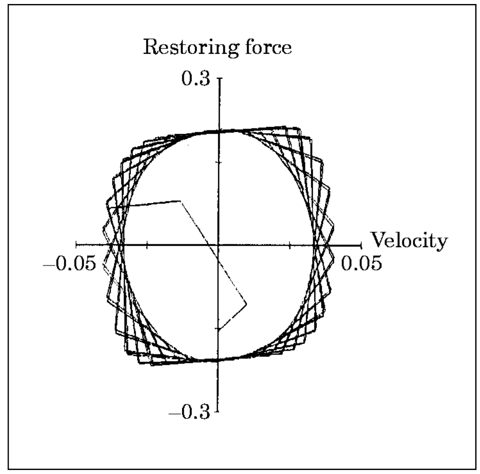

Figure 2. Measurements $g_2(\dot{y}_2)$.

restoring forces, and the relative errors of the minimum and maximum values. The simulated results indicate that the relative errors of the minimum and maximum values increase with the increase of the difference between the excitation amplitudes used in the training and validation phases. In the present example the relative difference of the excitation amplitudes used for training and validation is 5%. From Table 1 it can be calculated that the average relative errors of the minima and maxima of the measurements and identifications of displacements and velocities are 5·80% and 4·71%, respectively, and those of the measurements and identifications of restoring forces are 8·83% and 6·85%, respectively. In general, the average relative errors of restoring forces are larger than those of displacements and velocities, and all of these average relative errors have basically the same order of amplitude as the relative difference of the excitation amplitudes used for training and validation.

In order to inspect and compare the identification results, Figures (1) and (2) show the measurements $g_2(y_2)$ and $g_2(\dot{y}_2)$, where y_2 and $\dot{y}_2$ are taken as independent variables, respectively, Figures (3) and (4) show the corresponding identifications $g_2'(y_2')$ and $g_2'(\dot{y}_2')$,

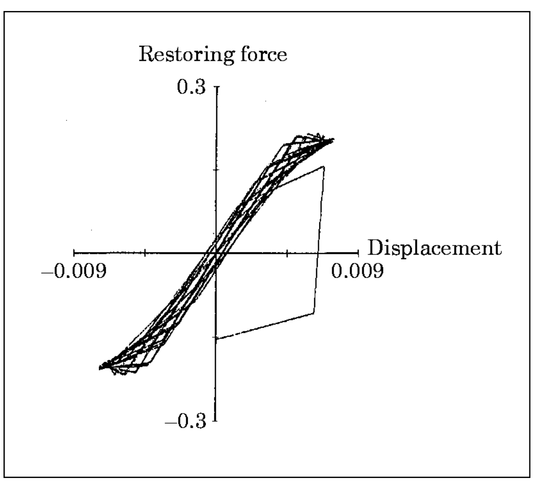

Figure 3. Identification results $g_2'(y_2')$.

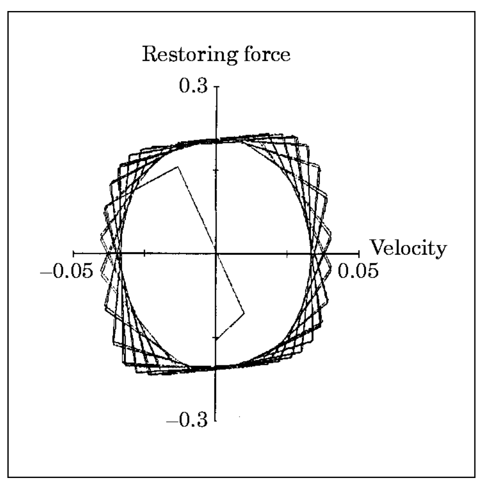

Figure 4. Identification results $g_2'(\dot{y}_2')$.

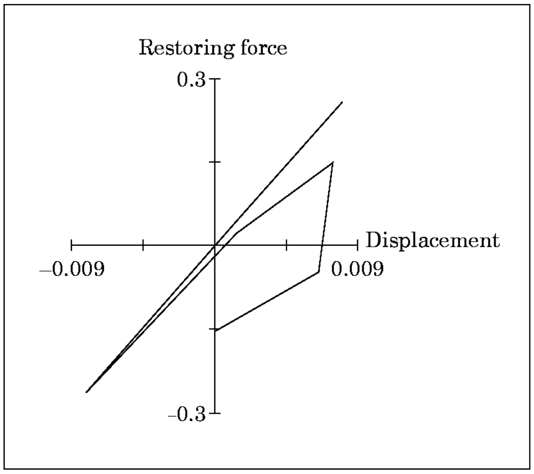

Figure 5. Measurements of linear system $g_2^*(y_2^*)$.

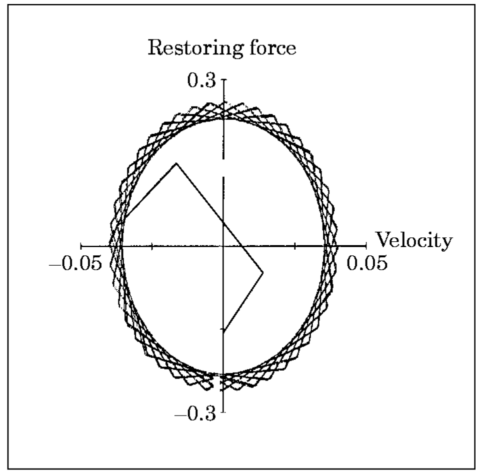

Figure 6. Measurements of linear system $g_2^*(\dot{y}_2^*)$.

respectively; Figures (5) and (6) show the restoring forces $g_2^*(y_2^*)$ and $g_2^*(\dot{y}_2^*)$, which are from the linear system corresponding to the non-linear system used in the present example.

From Table 1 and Figures 1–4 it can be seen that the procedure using neural networks and time domain measurements to identify the restoring forces is efficient for multi-degree-of-freedom vibration systems. The comparisons between Figures 3, 4 and Figures 5, 6 indicate that the existence of the non-linear terms in the system can be estimated from the identified restoring forces.

5. CONCLUSIONS

This study demonstrates that the procedure based on the use of artificial neural networks can be used effectively for the identification of the restoring forces of multi-degree-of-freedom non-linear vibration systems. The effect of non-linearities in the systems can be estimated from the identified results.

ACKNOWLEDGMENT

This work was partly supported by the Open Research Laboratory of Symbol Computation and Knowledge Engineering of the Chinese National Education Commission.

REFERENCES

1. S. F. Masri, A. G. Chassiakos and T. K. Caughey 1993 *American Society of Mechanical Engineers Journal of Applied Mechanics* **60**, 123–133. Identification of nonlinear dynamic systems using neural networks.
2. K. S. Mohammad, K. Worden and G. R. Tomlinson 1992 *Journal of Sound and Vibration* **152**, 471–499. Direct parameter estimation for linear and non-linear structures.
3. K. Worden and G. R. Tomlinson 1989 *Proceedings of the 7th International Modal Analysis Conference*, *Los Angeles*, 1347–1355. Application of the restoring force surface method to non-linear elements.
4. K. Worden 1989 *Ph.D Thesis*, *Mechanical Engineering Department*, *Heriot-Watt University*. Parametric and non-parametric identification of nonlinearity in structural dynamics.
5. D. E. Rumelhart and J. L. McClelland 1988 *Parallel Distributed Processing*: Explorations in the microstructure of cognition.